Broad-Band Cavity-Type Parametric Amplifier Design*

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Summary—**This paper tells how maximum bandwidth can be obtained from a nondegenerate parametric amplifier which utilizes a circulator. Expressions are derived for the gain bandwidth product and maximum possible gain bandwidth product. It is then shown how the Q of the cavities used for the signal and idler circuits may be kept at a minimum without degrading the noise performance of the amplifier. It is shown that best performance results when the TEM mode is used in coax, or, if waveguide is used, when the operating frequency is far away from the waveguide cutoff frequency. The diode used should have as high a self-resonant frequency as possible and the line admittance should be approximately the diode susceptance. Using a diode with a self-resonant frequency at the idler frequency will be seen to give optimum performance.**

This paper also discusses double tuning the signal circuit to achieve broader bandwidths. In this case, the addition of the second tuned circuit will be seen to give much broader bandwidths than one would expect from conventional filter theory.

Two sample amplifiers are considered and their bandwidths calculated. The effect of double tuning one of the amplifiers is then considered.

Introduction

THE PROBLEM of achieving optimum bandwidth is encountered in cavity-type parametric amplifiers designed to operate in the microwave region. Because the reactance diode has a finite Q and because of its lead inductance, there is a maximum theoretical bandwidth. Usually, however, the main difficulty in achieving maximum bandwidth is caused by the many frequency-dependent effects encountered in waveguide-type circuits.

This paper tells how maximum bandwith can be achieved by presenting an analysis of a nondegenerate parametric amplifier that utilizes a circulator. Expressions for the gain-bandwidth product and the maximum gain-bandwidth product are obtained. It is then shown how the microwave circuit may be adjusted to approach this gain-bandwidth product; that is, it is shown how the Q of the cavity used for signal and idler circuits may be kept at a minimum without degrading the noise performance of the amplifier. It is shown that best performance results when the TEM mode is used in coax, or, if waveguide is used, when the operating frequency is far away from the waveguide cutoff frequency. The diode used should have as high a self-resonant frequency as possible and the line susceptance should be approximately the diode susceptance. If a diode is chosen with a self-resonant frequency at the idler frequency, much broader bandwidths can be achieved in many cases because of the simple lumped-constant nature of the circuit.

This paper also discusses double tuning of the signal circuit to achieve broader bandwidths. In this, the addition of the second tuned circuit will be seen to give much broader bandwidths than one would expect from conventional filter theory. The reason for this is that the second tuned circuit alters the rate at which the idler reactance reduces the negative resistance. Two sample amplifiers will be considered and their theoretical bandwidth calculated. By careful application of the principles set forth in this paper, the bandwidth of almost any single-tuned parametric amplifier can be predicted quite closely, and an amplifier that approaches the maximum theoretical bandwidth can be designed.

Theoretical Bandwidth

The only amplifier considered is a reflection-type parametric amplifier using a circulator. The equivalent circuit for this amplifier is shown in Fig. 1, where the arrow indicates direction of energy flow. This type of amplifier is the most commonly used and provides low noise figures and stable operation.[1]

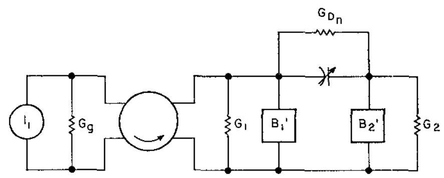

Fig. 1—Parametric amplifier equivalent circuit.

In deriving the expression for the gain-bandwith product, the general circuit relations derived by Bloom and Chang,[2] or Heffner and Wade,[3] will be used; the expression for the input admittance is the same as theirs except that it includes a diode loss conductance G_{D_n}. Throughout this paper the subscript n will always refer

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[1] R. C. Knechtli and R. D. Weglein, "Low-noise parametric amplifier," PROC. IRE, vol. 48, pp. 1218–1227; June, 1960.

[2] S. Bloom and K. K. N. Chang, "Theory of parametric amplifiers using nonlinear reactances," *RCA Rev.*, vol. 18, pp. 578–593; December, 1957.

[3] H. Heffner and G. Wade, "Gain, bandwidth, and noise characteristics of the variable-parameter amplifier," *J. Appl. Phys.*, vol. 29, pp. 1323–1331; September, 1958.

to the nth frequency. Thus, G_{D_1} and G_{D_2} are the diode conductances at the signal and idler frequencies, respectively. Since in a real diode the spreading resistance r_s is in series with the operating point capacitance C_0 and the lead inductance L, then the diode loss conductance and effective capacitance C_n' are frequency dependent and, in fact, given as

$$G_{D_n} = \frac{1/r_s}{1 + Q_{D_n}^2\left(\frac{\omega_n^2}{\omega_D^2} - 1\right)^2}$$

$$C_n' = \frac{C_0 Q_{D_n}^2}{1 + Q_{D_n}^2\left(\frac{\omega_n^2}{\omega_D^2} - 1\right)^2}, \tag{1}$$

where

$\omega_D = \frac{1}{\sqrt{LC_0}}$ is the diode self-resonant frequency, and

$Q_{D_n} = \frac{1}{\omega_n C_0 r_s}$ is the diode Q at the frequency ω_n.

The normalized input admittance to the amplifier is then given as

$$y = \frac{G_c}{G_g} - \frac{G_N}{G_g}\left(1 + \frac{B_2^2}{G_{T_2}^2}\right)^{-1} + j\left[\frac{B_1}{G_g} - \frac{G_N B_2}{G_g G_{T_2}}\left(1 + \frac{B_2^2}{G_{T_2}^2}\right)^{-1}\right], \tag{2}$$

and the reflection voltage gain Γ by

$$\Gamma = \frac{1-y}{1+y}. \tag{3}$$

The following definitions are useful:

$G_c = G_1 + G_{D_1}$ = signal circuit admittance,
$G_{T_2} = G_2 + G_{D_2}$ = idler circuit admittance,
$G_N = \omega_1\omega_2 C_1' C_2' \gamma^2 / G_{T_2}$ = negative conductance,
$\gamma = C_1/2C_0$ = Fourier capacitance ratio,
$B_1 = B_1' + \omega C_0$ = total signal circuit susceptance,
$B_2 = B_2' + (\omega_c - \omega)C_0$ = total idler circuit susceptance,

$$\xi = \frac{G_N}{G_g} - \frac{C_c}{G_g},$$

$\Gamma_0 = \frac{1+\xi}{1-\xi}$ = resonance voltage gain,

$G_0 = \Gamma_0^2$ = resonance power gain.

$$\xi = \frac{\sqrt{G_0} - 1}{\sqrt{G_0} + 1},$$

where B_1' and B_2' consist of an ideal filter and the susceptance element necessary to resonate the signal and idler circuit, respectively.

In order to compute the gain-bandwidth product, we will make the following definitions and approximations:

$$\delta_1 = \frac{\omega}{\omega_1} - \frac{\omega_1}{\omega} \approx \frac{2(\omega - \omega_1)}{\omega_1},$$

$$\delta_2 = \frac{\omega_3 - \omega}{\omega_2} - \frac{\omega_2}{\omega_3 - \omega} \approx -\frac{\omega_1}{\omega_2}\delta_1,$$

Q_c = unloaded signal circuit tank Q,

Q_2 = idler tank Q,

$$Q_T = Q_c\frac{G_c}{G_g} + \frac{Q_2 G_N \omega_1}{G_g \omega_2},$$

$$B_1/G_c = Q_c\delta_1,$$

$$B_2/G_{T_2} = Q_2\delta_2 = -\left(\frac{\omega_1}{\omega_2}Q_2\delta_1\right).$$

The approximations are valid to within 10 per cent for bandwidths less than 40 per cent if it is assumed that the susceptance variation, as a function of frequency, is linear over the range of interest. We will assume this to be the case since, as will be shown later, it is not possible to obtain bandwidths much greater than about 2.5 per cent for a single-tuned circuit when operating at 20-db gain with good noise figure.

Under normal operation with good noise figures, the following may also be assumed:

$$\left(\frac{G_c}{G_g}Q_c + \frac{\omega_1 G_N}{\omega_2 G_g}Q_2\right) \gg \frac{G_c\omega_1^2}{G_g\omega_2^2}Q_c Q_2^2\delta_1^2$$

$$G_0 - 2 \approx G_0;$$

and that $\zeta \ll 1$, where

$$\zeta = \frac{4\omega_1^4 Q_2^4}{\omega_2^4(1+\xi)^2}\left[G_0\left(1 + \frac{G_c}{G_g}\right)^2 - 2\left(1 - \frac{G_c}{G_g}\right)^2\right] \cdot \left\{\frac{Q_T^2 G_0}{(1+\xi)^2} + \frac{2Q_2^2\omega_1^2}{(1+\xi)\omega_2^2}\left[\sqrt{G_0}\left(1 + \frac{G_c}{G_g}\right) - 2\left(1 - \frac{G_c}{G_g}\right)\right]\right\}^{-2}.$$

Let $\Delta\omega = 2(\omega - \omega_1)$ at the half-power points; the gain bandwidth product may now be written

$$\frac{\sqrt{G_0}\,\Delta\omega}{\omega_1} = \left[\frac{Q_T^2}{(1+\xi)^2} + \frac{2\omega_1^2 Q_2^2[\sqrt{G_0}\,(1 + G_c/G_g) - 2(1 - G_c/G_g)]}{\omega_2^2(1+\xi)G_0}\right]^{-1/2}. \tag{4}$$

Examination of (4) reveals that it consists of two parts: one contains the term Q_T^2, due only to the reactive part of the input admittance; the other part contains the term Q_2^2, due only to the drop in negative conductance of the input admittance. It is the latter term that is neglected by Heffner and Wade, and Bloom and Chang. This term can only be neglected for gains greater than 30 db. Usually, however, stable operation is nearer 20 db, and this term cannot be neglected.

At this point, it is desirable to determine the magnitude of G_c/G_g and G_N/G_g. The quantity G_N/G_g is related to the gain and G_c/G_g by

$$\frac{G_N}{G_g} = \frac{G_c}{G_g} + \frac{\sqrt{G_0} - 1}{\sqrt{G_0} + 1} \cdot \qquad (5)$$

Thus, it is only necessary to determine G_c/G_g. The determining factor on G_c/G_g is the noise figure of the parametric amplifier. At high gains, this is given as

$$F = \left(1 + \frac{\omega_1}{\omega_2}\right)\left(1 + \frac{G_c}{G_g}\right).^{4,5} \qquad (6)$$

For good noise figures the generator is to be overcoupled as much as possible to the signal circuit. Overcoupling is limited by the diode, since more pump power is required to achieve gain as the generator conductance is increased. The limit on the over-coupling has already been discussed by Knechtli and Weglein[1] and will not be repeated here. This is given by

$$\left(\frac{G_g}{G_c}\right)_{\max} = \left(\left(\gamma^2 Q_{D_1}{}^2 \frac{\omega_1}{\omega_2}\right) - 1\right)\left(\frac{\sqrt{G_0} + 1}{\sqrt{G_0} - 1}\right), \qquad (7)$$

in which it is assumed that all loss is in the diode conductance. Since both (6) and (7) contain G_c/G_g and ω_1/ω_2, there is an optimum frequency ratio and coupling ratio for minimum noise figure. These are[6]

$$\left(\frac{G_g}{G_c}\right)_{\text{opt}} = \left(\sqrt{1 + \gamma^2 Q_{D_1}{}^2}\right)\left(\frac{\sqrt{G_0} + 1}{\sqrt{G_0} - 1}\right) \qquad (8)$$

and

$$\left(\frac{\omega_1}{\omega_2}\right)_{\text{opt}} = \left(1 + \sqrt{1 + \gamma^2 Q_{D_1}{}^2}\right)/\gamma^2 Q_{D_1}{}^2. \qquad (9)$$

[4] The exact expression for a reflection-type amplifier which gives the gain variation is

$$F = 1 + \frac{(\sqrt{G_0} + 1)^2}{G_0}\left[\frac{G_c}{G_g} + \frac{\omega_1}{\omega_2}\left(\frac{\sqrt{G_0} - 1}{\sqrt{G_0} + 1} + \frac{G_c}{G_g}\right)\right].$$

[5] H. Heffner and G. Wade, "Minimum noise figure of a parametric amplifier," *J. Appl. Phys.*, vol. 29, p. 1262; August, 1958.

[6] K. Kotzebue, "Optimum noise performance of parametric amplifiers," Proc. IRE, vol. 48, pp. 1324–1325; June, 1960.

Usually, γ can be made about 0.29 as may be determined by solving the hypergeometric series for the capacitance coefficients.[7]

Notice here that to obtain a noise figure of less than 3 db, it is required that $\omega_1/\omega_2 = 1/2$, $G_c/G_g = 1/2$, and that the diode have a Q of at least 8.

Now let us determine the absolute maximum possible gain-bandwidth product for a single-tuned circuit. To achieve resonance, the simplest signal or idler circuit must have a diode and a reactive element. Such a circuit is shown in Fig. 2. The X indicates the reactance which must be added to achieve resonance. When $\omega_D < \omega_n$, the circuit Q may be written as $Q = Q_{D_n}(\omega_n{}^2/\omega_D{}^2)$; when $\omega_D > \omega$, it may be written as $Q = Q_{D_n}$. Thus the minimum obtainable Q is limited by the diode Q and self-resonant frequency. Obviously, a diode should be chosen with as high a self-resonant frequency as possible. It will be assumed that both signal and idler circuits are resonated independently through use of an ideal filter,[8] and that all the loss occurs in the diode conductance. Thus, when both signal and idler frequencies lie below the diode self-resonant frequency, the maximum gain-bandwidth product is

$$\frac{\sqrt{G_0}\Delta\omega}{\omega_1} = \left(\frac{Q_{D_1}}{1 + \xi}\right)^{-1} \left\{\frac{G_c}{G_g} + \frac{G_N \omega_1{}^2}{G_g \omega_2{}^2} + \frac{2\omega_1{}^4(1 + \xi)}{\omega_2{}^4 G_0} \cdot \left[\sqrt{G_0}\left(1 + \frac{G_c}{G_g}\right) - 2\left(1 - \frac{G_c}{G_g}\right)\right]\right\}^{-1/2}. \qquad (10)$$

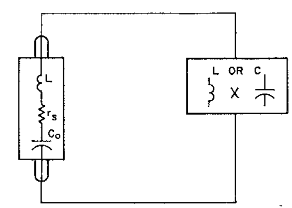

Fig. 2—Simplest parametric signal or idler resonant circuit.

In Fig. 3, the gain-bandwidth product normalized to diode Q is plotted against G_c/G_g for various idler-to-signal-frequency ratios to show the importance of the coupling factor, G_c/G_g, on bandwidth. Since the parameters in (10) all enter into the noise figure expression to maintain a noise figure less than 3 db, it is possible to obtain at best a 2.5 per cent bandwidth at 20-db gain for a single-tuned circuit. In addition, most double-tuned circuits will not have bandwidths much in excess of 2.5 per cent and certainly less than 25 per cent at 20-db gain. Thus, the approximations used in this paper will be valid.

[7] S. Sensiper and R. D. Weglein, "Capacitance and charge coefficients for parametric diode devices," *J. Appl. Phys.*, to be published.

[8] Usually it is not possible to build an amplifier whose filters are ideal and do not affect the bandwidth of an amplifier. However, if the idler frequency is much greater than the signal frequency, a filter can be built whose dimensions are sufficiently small at the signal frequency so that its performance approaches that of an ideal filter.

Achieving Optimum Q in a Microwave Circuit

It is now convenient to consider how Q_c and Q_2 may be determined for a diode mounted in a microwave circuit, and what can be done to achieve the minimum possible Q. The Q of any circuit for reasonable frequency changes can be expressed in differential form as[9]

$$Q = \frac{\omega}{2R}\frac{\partial X}{\partial \omega} \quad \text{or} \quad Q = \frac{\omega}{2G}\frac{\partial B}{\partial \omega}. \tag{11}$$

Thus,

$$Q_c = \frac{\omega_1}{2G_c}\frac{\partial B_1}{\partial \omega} \quad \text{and} \quad Q_2 = \frac{\omega_2}{2G_{T_2}}\frac{\partial B_2}{\partial \omega}. \tag{12}$$

There are two ways in which a diode may be mounted in a microwave circuit: in series or in parallel with a transmission line. Usually, a shorting plunger is used to achieve resonance; however, the Q we wish to consider is that seen from the terminals of the capacitance, since the susceptance transfer occurs across these terminals. Because of this, the equivalent circuit for either the series or parallel mount will be that shown in Fig. 4.

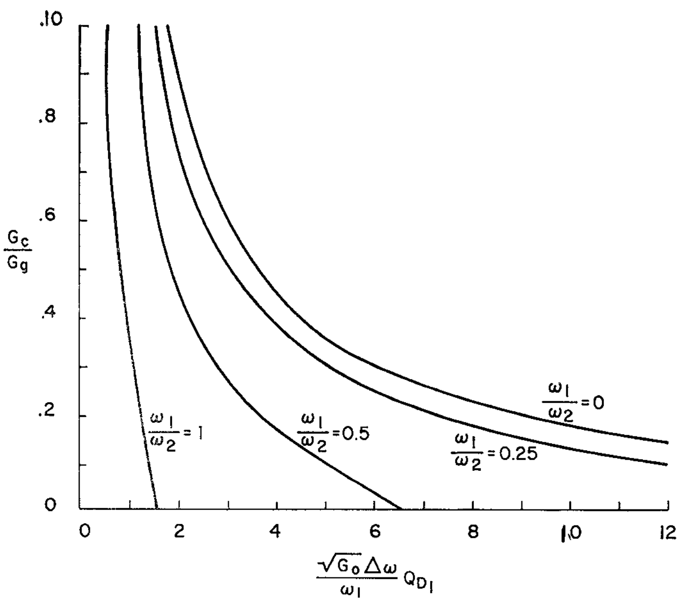

Fig. 3—Gain-bandwidth product vs input coupling for various signal-to-idler-frequency ratios. Both signal and idler frequencies are below the diode self-resonant frequency.

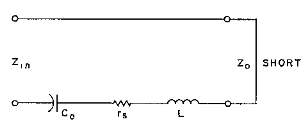

Fig. 4—Equivalent circuit of a diode mounted in series in a transmission line.

[9] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 8, p. 230; 1948.

The inductance shown is the lead inductance. Let us consider the series mount first.

The most common example of a series-mounted diode is that mounted in the center conductor of a coaxial line. In this, the characteristic line impedance Z_0 and the wavelength are not frequency-dependent in any special way as they would be in waveguide, unless the TEM mode is not used. It will be assumed that the series mount is in coaxial line and that the TEM mode is used. It will also be assumed that the losses in the circuit are all in the diode, and that other circuit losses, such as cavity-wall losses, are negligible. This is a good assumption if there are no lossy elements (*e.g.*, filters) present; however, if other lossy elements are present, their effective resistance must be added to the diode series resistance. The admittance of Fig. 4 for the series mount is given as

$$Y = j\omega C_0 + \frac{1}{r_s + j(\omega L + Z_0 \tan\theta\omega)}, \tag{13}$$

where $\theta = l/c$, and where l is the distance to the short. All discontinuity capacitances and inductances are assumed negligible. For a reasonably high-Q circuit ($Q>3$) the admittance may be simplified as

$$Y = \frac{r_s}{[\omega L + Z_0\tan\theta\omega]^2} + j\left[\omega C_0 - \frac{1}{\omega L + Z_0 \tan\theta\omega}\right]. \tag{14}$$

The Q of this circuit may then be expressed as

$$Q = \frac{\omega}{2G}\frac{\partial B}{\partial\omega} = \frac{Q_{D_n}}{2}\left[1 + \frac{\omega_n^2}{\omega_D^2} + \frac{2\theta\omega_n}{\sin 2\theta\omega_n}\left(1 - \frac{\omega_n^2}{\omega_D^2}\right)\right]. \tag{15}$$

Upon examination of (15), the conditions imposed on $\theta\omega_n$ and, thereby, on Z_0 can be determined in order to achieve the smallest value for Q possible. In (15) these factors enter as $2\theta\omega_n/\sin 2\theta\omega_n$. The minimum values are solutions of the equation, $\tan 2\theta\omega_n = 2\theta\omega_n$. However, to resonate the circuit, $\theta\omega_n$ is restricted to certain ranges of values, depending on the value of ω_D; that is, for

$$\omega_D > \omega, \; 0 < \theta\omega_n < \pi/2$$

and for

$$\omega_D < \omega, \; \pi/2 < \theta\omega_n < \pi.$$

Now, the first two minimums occur for $2\theta\omega_n = 0$ and $2\theta\omega_n = 257.5°$. Successive minimums occur approximately for odd multiples of $\pi/2$. In the case of the first minimum, which occurs for $\omega_D > \omega$, the short is to be placed as close to the diode as possible. This corresponds to making Z_0 as large as possible. For $\omega_D < \omega$, the short is to be placed at $\theta\omega_n = 128.2°$, which corresponds to $\tan\theta\omega_n = 1.23$; or, Z_0 is to equal 0.81 times the diode reactance.

This corresponds to almost matching the diode reactance. The other minimum further down the line corresponds more closely to matching the diode reactance to Z_0. Of course, for the smallest Q the short should be placed at the minimum closest to the diode. When operating at the optimum values for $\theta\omega_n$, the circuit Q's for the following special cases are

$$\omega_D \gg \omega_n: \; Q = Q_D$$
$$\omega_n \gg \omega_D: \; Q = Q_D(\omega_n^2/\omega_D^2)2.8. \tag{16}$$

It is obvious from (15) and (16) that it is desirable to have ω_D as high as possible. Evidently, in this type of circuit unless $\omega_D > \omega_n$, the Q of the circuit does not equal the minimum obtainable Q. However, improvement in bandwidth can be realized by properly selecting Z_0.

The most common example of a diode mounted in parallel with a transmission line is one which is mounted across a waveguide. Here, it is necessary to consider the frequency variations of Z_0 and $\theta\omega_g$. Assuming again that we have a reasonably high-Q circuit, the admittance may be expressed as

$$Y = \frac{r_s}{(\omega L + Z_0 \tan \theta\omega_g)^2} + j\left[\omega C_0 - \frac{1}{\omega L + Z_0 \tan \theta\omega_g}\right]; \tag{17}$$

and for TE modes,

$$Z_0 \approx (2b/a)\sqrt{\mu/\epsilon}\,(1 - \omega_{cg}^2/\omega^2)^{-1/2}, \tag{18}$$

where a is the waveguide width, b its height, and

$$\omega_g = \omega(1 - \omega_{cg}^2/\omega^2)^{1/2}, \tag{19}$$

in which ω_{cg} is the waveguide cutoff frequency.

Although Z_0 is not clearly defined for waveguide, if the diode is mounted in the center of the waveguide, the three possible definitions for Z_0 will differ only a small amount. Also, the optimum value for Z_0 will be seen not to be critical, so that a variation of 30 per cent in Z_0 will not appreciably affect the determined value for bandwidth. Again it has been assumed that all other capacitance or inductances are negligible.

The Q of the circuit is

$$Q = \frac{Q_{D_n}}{2}\left\{\left(\frac{\omega_n^2}{\omega_D^2} + 1\right) + \frac{\left(1 - \dfrac{\omega_n^2}{\omega_D^2}\right)}{\left(\dfrac{\omega_n^2}{\omega_{cg}^2} - 1\right)} \left[\frac{2\theta\omega_g}{\sin 2\theta\omega_g}\left(\frac{\omega_n^2}{\omega_{cg}^2} - 1\right)^{1/2} \frac{\omega_n}{\omega_{cg}} - 1\right]\right\}. \tag{20}$$

Since Q_D, ω_D, and ω_n are fixed, the only variables are ω_{cg} and $2\theta\omega_g/\sin 2\theta\omega_g$. For minimum Q, $\omega_{cg} \ll \omega_n$ and $2\theta\omega_g/\sin 2\theta\omega_g$ are to be kept at a minimum. As before, in order to achieve resonance, limits are imposed on $\theta\omega_g$. These are the same as in the previous case; *i.e.*, for $\omega_D > \omega$: $0 < \theta\omega_g < \pi/2$, and for $\omega_D < \omega$: $\pi/2 < \theta\omega_g < \pi$. Thus, when $\omega_D > \omega$ the short is to be placed as close to the diode as possible and Z_0 is to be correspondingly large. When $\omega_D < \omega$, the short is to be placed at the next minimum, which occurs for $\theta\omega_g = 128.2°$; and Z_0 is to be made equal to 0.81 times the diode susceptance. In Fig. 5, Q/Q_{D_n} is plotted vs ω_{cg}/ω for $\omega_D = 2$ and for $\omega_D = 1/2$. It is assumed that Q_D is much greater than 1. Note that in some cases operation near the guide cutoff frequency raises the circuit Q considerably. Note also that this type of circuit only approaches the minimum theoretical Q when $\omega < \omega_D$, and when the operating frequency is far from the guide cutoff frequency. The only other way to lower the effective Q of the circuit is to double tune the signal circuit. Double tuning to increase gain-bandwidth product will be discussed in the next section.

When operating with $\omega = \omega_D$ at the diode self-resonant frequency, it may be observed that the Q of the circuit becomes merely the diode Q. A very broad-band amplifier can be built by using this fact. A particular example

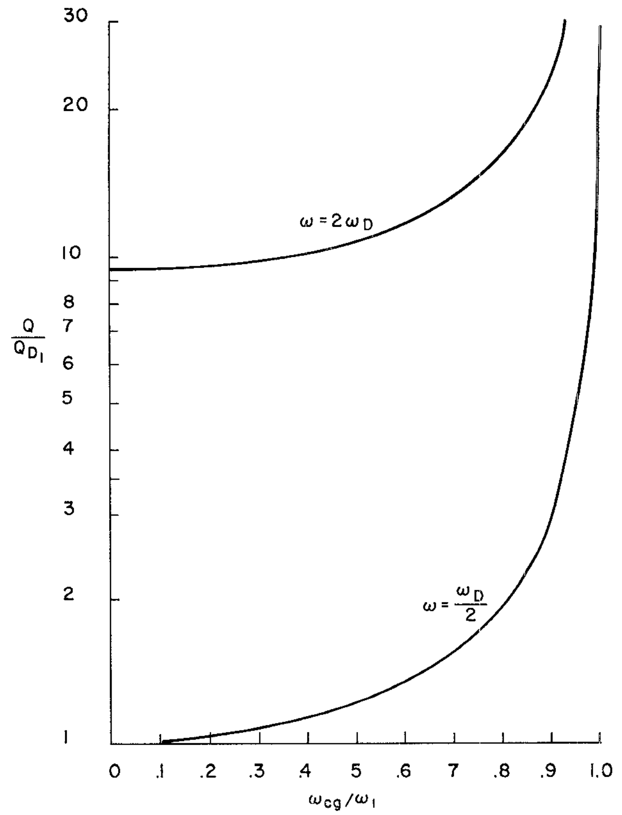

Fig. 5—Circuit Q normalized to diode Q vs ratio of guide cutoff frequency to signal frequency for TE modes in rectangular guide.

of this would be to have the diode used with its self-resonant frequency at the idler frequency, or in some cases, it may be desirable to add a small capacitance or capacitive discontinuity in series with the diode to raise its resonance frequency to the idler frequency. Examples of these applications are shown in Fig. 6. The idler frequency is suggested because its frequency most commonly limits the bandwidth, and the idler frequency could be adjusted to maximize bandwidth simply by changing the pump frequency. Also, by reducing the idler Q, the second term in (4) is considerably reduced. Operating with a diode which has a self-resonance at the idler could improve the bandwidth by a factor of 3 or more.

Double-Tuned Circuit Bandwidths

An alternate way to improve the bandwidth is to double tune the signal circuit. Briefly, to describe the theory:[10,11] Consider the signal circuit of a double-tuned amplifier such as shown in Fig. 7. The admittance at plane 1 is given by (2). With the same assumptions used

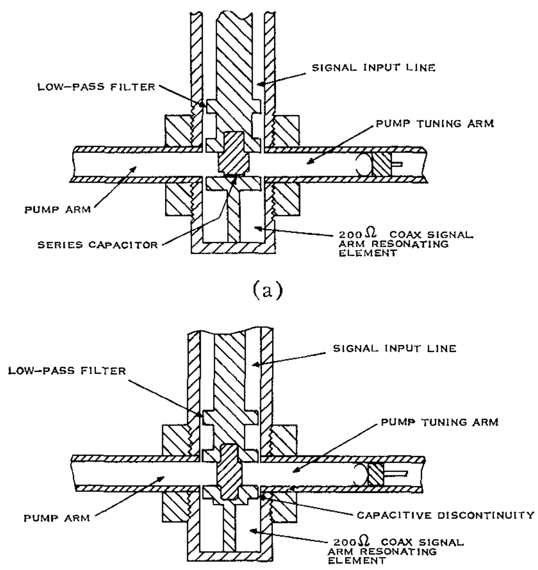

Fig. 6—Circuits with series idler resonance.

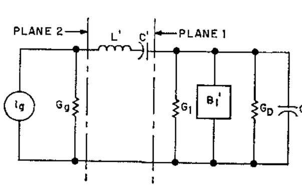

Fig. 7—Double-tuned signal circuit equivalent circuit.

[10] H. Seidel and G. F. Hermann, "Circuit aspects of parametric amplifiers," 1959 WESCON Convention Record, pt. 2, pp. 83–90.

[11] G. L. Matthaei, "A Study of the Optimum Design of Wide-band Parametric Amplifiers and Up Converters," presented at the PGMTT Natl. Symp., San Francisco, Calif.; May 9–11, 1960.

to write (4), and assuming $G_c \ll G_g$, (2) may be rewritten as

$$y_1 = \frac{-\xi + jQ_T\delta}{1 + \epsilon^2\delta^2}, \tag{21}$$

where $\epsilon = (\omega_1/\omega_2)Q_2$.

The impedance at plane 2 is

$$z_1 = \frac{1 + \epsilon^2\delta^2 - Q_T\beta\delta^2 - j\beta\xi\delta}{-\xi + jQ_T\delta}, \tag{22}$$

where $\beta = \omega_1 L' G_g = Q$ of the second cavity, loaded only by the generator. The power gain may now be calculated as

$$\frac{G(\omega)}{G_0} = \frac{\left[1 + \dfrac{(\epsilon^2 - Q_T\beta)\delta^2}{1+\xi}\right]^2 + \dfrac{\delta^2}{(1+\xi)^2}(\beta\xi + Q_T)^2}{\left[1 + \dfrac{(\epsilon^2 - Q_T\beta)\sqrt{G_0}\,\delta^2}{1+\xi}\right]^2 + \dfrac{\delta^2 G_0}{(1+\xi)^2}(\beta\xi - Q_T)^2}.$$

The solution of an equation in δ^2 and δ^4 is obtained. For maximally flat response the coefficient of the δ^2 term is set equal to zero. This fixes the value for β. For reasonably high gains ($G_0 > 15$ db), the value for β is given approximately by

$$\beta \approx \frac{Q_T}{\xi}\left[1 + \frac{4}{G_0} + \frac{2(\sqrt{G_0} - 1)}{G_0}\right] \cdot \left[1 \pm \sqrt{1 - \frac{1 + \dfrac{4(\sqrt{G_0} - 3)\epsilon^2}{G_0 Q_T^2}}{\left[1 + \dfrac{4}{G_0} + \dfrac{2(\sqrt{G_0} - 1)}{G_0}\right]^2}}\right]. \tag{24}$$

The equation for the gain-bandwidth product is

$$\frac{\Delta\omega\sqrt{G_0}}{\omega_1} = \left[\frac{(1+\xi)^2 G_0^2}{(\epsilon^2 - Q_T\beta)^2}\right]^{\frac{1}{4}}, \tag{25}$$

which at high gains is

$$\frac{\sqrt{G_0}\,\Delta\omega}{\omega_1} = \frac{(4G_0)^{\frac{1}{4}}}{|\epsilon^2 - Q_T^2|^{\frac{1}{2}}}, \tag{26}$$

so that there is an improvement for a double-tuned signal circuit.

It is possible for the denominator of (25) to go to zero in some special cases. In any case there is considerable improvement in the gain-bandwidth product because β is approximately equal to Q_T and ϵ is just slightly smaller than Q_T. The factor ϵ differs from Q_T by just G_cQ_c/G_g. Thus, G_cQ_c/G_g is to be made as small as possible. Of course it is not possible to achieve infinite gain-bandwidths since, among other assumptions, approximately linear frequency dependence is assumed for the reactance variation. However, (25) predicts a greater bandwidth improvement than would a conventional filter. Conventional filter theory usually assumes that the amplifier frequency response is determined by just the imaginary part of the input impedance. This is not a good assumption for a parametric amplifier unless the idler Q is less than the loaded signal circuit Q.

In the following section two amplifiers will be designed and their bandwidths computed.

Theoretical Broad-Band Amplifier Designs

The first amplifier to be designed is similar to the one shown in Fig. 6(b), though without the capacitive discontinuity. The diode available has a self-resonant frequency at 12.6 kMc. The amplifier then has the following characteristics:

$$f_1 = 5000 \text{ Mc},$$
$$f_2 = 12.6 \text{ kMc},$$
$$f_3 = 17.6 \text{ kMc},$$
$$f_1/f_2 = 0.396,$$
$$f_c/f_1 = 26.4,$$
$$\gamma = 0.29,$$
$$G_g/G_c = (\text{calculated})\ 27.1.$$

The signal circuit is formed in a 200-ohm coax line. The line impedance is chosen large to obtain the minimum possible Q. Since the signal frequency is considerably below the diode self-resonant frequency, an inductive length is added to achieve resonance. A one-section filter one-fourth wavelength long at the idler frequency is employed to provide a short circuit to the idler, but to pass the signal. Assuming the impedance of the line at this filter section is 10 ohms, then the effective line impedance at the signal frequency would correspond approximately to a $\sqrt{150 \times 10} = 44.7$-ohm line. The short for the signal frequency then is about $\pi/8$ away from the diode. The pump arm is formed in a waveguide which is beyond cutoff for the idler and signal frequencies. A band-pass filter in the signal arm places a short circuit at the waveguide wall at the idler frequency. The signal arm is tuned by placing the short behind the diode at the proper place.

Using this information, the circuit Q's and gain-bandwidth product may be computed as

$$Q_c = \frac{26.4}{2}\left[1 + \left(\frac{1}{13}\right)^2 + \frac{2\pi/4}{\sin 2\pi/4}\left(1 - \frac{1}{13}\right)^2\right]$$
$$= 33.8,$$
$$Q_2 = Q_{D_2} = 10.5,$$
$$Q_T = 4.80,$$
$$\sqrt{G_0}\Delta f = 1700 \text{ Mc},$$
$$\Delta f_{20\,\text{db}} = 170 \text{ Mc}.$$

This bandwidth is to be compared with a bandwidth of 12.8 Mc which would have been achieved had a diode with a self-resonance at 3 kMc been used with the idler circuit formed in waveguide. Some improvement over 12.8 Mc could be achieved by moving the signal-circuit filter up so that the idler resonated in the coax. In this case, however, the frequency dependence of the line must still be taken into account.

The second amplifier to be designed is shown in Fig. 8 and employs double tuning in the signal circuit to provide increased bandwidths. The amplifier has the following characteristics:

$$f_1 = 9000 \text{ Mc},$$
$$f_3 = 35 \text{ kMc},$$
$$f_1/f_2 = 0.375,$$
$$f_D/f_2 = 0.300,$$
$$f_{cq}/f_2 = 0.725,$$
$$f_{cg}/f_1 = 0.585,$$
$$\gamma = 0.29,$$
$$f_c/f_1 = 21.0,$$
$$G_g/G_c = 15.8.$$

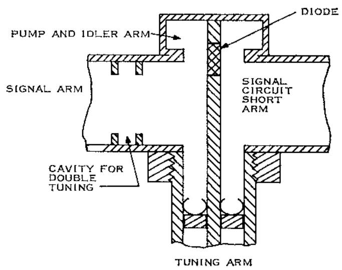

Fig. 8—Double-tuned signal circuit.

The signal arm is in reduced-height waveguide of about 50-ohm impedance to obtain a small circuit Q. The pump and idler occur in the same guide, reduced to have an impedance of 50 ohms, again to obtain a small circuit Q.

To determine the Q of the second cavity, it is necessary to compute the Q's of the single-tuned amplifier. These are

$$Q_c = 61.8,$$
$$Q_2 = 48.4,$$
$$Q_T = 19.93,$$
$$\epsilon = 18.1.$$

For a single-tuned circuit this would correspond to a bandwidth of 73 Mc at 20-db gain. For double tuning, the second-cavity Q is calculated from (24) to be 8.68. This then gives a bandwidth of 310 Mc, which is an improvement of a factor of 4.25.

Experimental Results

Calculations of the bandwidth of single-tuned L- and S-band parametric amplifiers constructed at Texas Instruments and elsewhere agree with the experimentally determined bandwidth. The amplifiers were all of the type shown in Fig. 8, except that the discontinuity capacitance was omitted. An S-band amplifier constructed at Texas Instruments which originally had 10-Mc bandwidth was made to have 70-Mc bandwidth at 16-db gain by operating with the diode self-resonance at the idler frequency. In this amplifier, both the signal and idler frequencies were below cutoff in the pump waveguide instead of using cutoff filters. A C-band amplifier constructed elsewhere was of the type shown in Fig. 6(b). This particular amplifier had as much as a 75-Mc bandwidth at 20-db gain, and gave 12- to 30-db gain over a 200-Mc bandwidth. The gain response was not maximally flat over the 200-Mc range, however.

In general, it may be said that broad bandwidths are achieved when a diode series resonance is achieved at the idler frequency. All resonant circuits should be formed as close to the diode as possible to avoid high-Q-type circuits. There is no exact method for achieving an idler resonance; ordinarily, this can be determined experimentally.

Design Comments

In some cases it may not be desirable to obtain the minimum circuit Q. In particular, if one seeks to obtain the minimum circuit Q, the series resistance or conductance may become so large that, to over-couple the generator to this resistance or conductance, it may be necessary to double-tune the signal circuit to achieve the proper coupling at resonance. In order to avoid this problem, since the signal circuit has such a broad bandwidth already, it may be desirable not to use the line impedance which corresponds to minimum Q.

Filters should be placed in a low-impedance line wherever possible, since a 0.4-db-loss filter in a 50-ohm line corresponds to an additional series resistance of 1.0 ohm, while the same loss in a 15-ohm line corresponds to only a series resistance of 0.3 ohm.

In many cases the shunt capacitance of a diode will raise its effective circuit Q, although the γQ_{D_n} product remains constant. This effect must be considered when computing bandwidth in a practical circuit. Therefore, for a diode with a shunt capacitance of C_s across the series resistance r_s and operating point capacitance C_0, the effective Q becomes

$$Q_{D_n \text{ effective}} \approx Q_{D_n}\left(1 + \frac{C_s}{C_0}\right).$$

Thus, it is desirable to choose a diode with the lowest possible shunt capacitance.

Conclusions

Optimum gain-bandwidth product can be achieved if the characteristic line impedance is adjusted so as to obtain the minimum circuit Q. Substantial improvement can be achieved if the signal circuit is double tuned or if a diode series resonance is obtained at the idler frequency. When operating in waveguide, the operating frequency should be as far from the waveguide cutoff frequency as possible. By careful application of the principles set forth in this paper, one is able to predict bandwidth performance and design an amplifier with optimum gain-bandwidth product.

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